REVIEWS

The Gulf Stream: A Physical and Dynamical Description. By HENRY STOMMEL. University of California Press, 1959. 202 pp. 45s.

The intense current system in the western North Atlantic Ocean, called the Gulf Stream, is perhaps the best explored of all oceanic phenomena, although improving technique and instrumentation for research at sea are continually revealing features of major interest of the stream. The existence of the current has long been known from the directly observable surface velocities, and from the distinctly different properties, e.g. temperature, of the water masses on either side of the stream. The velocity structure and transport of the current have been inferred from observable profiles of density. From the Straits of Florida north to Cape Hatteras the current follows the coast of the United States; at that point it leaves the coast and flows to the north-east, holding together at least to the region of the Grand Banks. The stream is of the order of 100 km in width, and does not broaden appreciably after leaving the coast, although the position of the stream does become considerably more variable. In the region away from the coast, meanders (or loops in the stream) occur on a length scale of hundreds of kilometres (the downstream scale of the current itself is thousands of kilometres). In 1950, a multiple ship survey, lasting for 17 days, observed a meander to grow, to oscillate and finally to detach completely as an elongated eddy.

An intense western current system is not peculiar to the North Atlantic. Analogous currents are known to exist in the western parts of all oceans bounded by land masses which run roughly north to south. Most notable is the Kuroshio of the North Pacific. Moreover, the western current is not an isolated or local phenomenon, but is more appropriately described as the most striking feature of the total large scale current systems, i.e. of the so-called general circulation of an ocean.

The general oceanic circulation poses a highly complex problem of fluid mechanics, a problem which most fluid dynamicists will find somewhat unusual because of the unique manner in which the motion relative to the rotating earth is constrained by Coriolis accelerations. The accelerations due to the component of the earth's rotation parallel to gravity completely dominate those due to the normal component. Thus Coriolis effects vary from practically zero at the equator to a maximum at the poles. Correspondingly, there is a differential advection of vorticity relative to the earth by the basic rotational motion, which, over the main body of the ocean, almost entirely balances the curl of internal stresses caused by the atmospheric winds acting upon the upper free surface. The field of motion resulting from this balance has, however, a meridional component which is everywhere towards the equator. Since mass conservation does not permit a net transport across the ocean basin, a region in which some other process of vorticity transfer becomes dominant is necessary to complete the flow field. Because of the basic asymmetry of the rotational constraint, this singular region of intense poleward flow occurs along the western boundary of the ocean. In a simple but elegant paper in 1948, the author of *The Gulf Stream* first showed the existence of the intense western current system to be intimately related to the latitudinal variation of Coriolis effects. This initiated a period of research and study which during the intervening years has produced sufficient understanding of the Stream to make this book feasible.

The stated purpose of the book is to communicate to physical scientists in general what is known about the intriguing natural phenomenon with which it deals. The chapter headings provide a good indication of the author's approach to this task. They are as follows: Historical introduction, Methods of observation, The geostrophic relationship, Large-scale features of the North Atlantic circulation, The wind system over the North Atlantic, Linear theories of the Gulf Stream, Non-linear theories of the Gulf Stream, Meanders in the Stream, Fluctuations in the currents, Role of the thermohaline circulation. The manner and level of discussion of the different topics vary throughout the book. The brief but scholarly historical section will be most enjoyable to those whose understanding of a subject is enhanced by a knowledge of the manner in which presently accepted ideas have evolved. The treatment of factual information and very basic theoretical ideas which follows is lucid and self-contained, and thus invaluable to the non-specialist. A higher level of mathematical sophistication is necessary for a full understanding of the theories of the mean current which are next presented in some detail.

The remaining chapters on time-dependent properties and the thermal circulation, which deal with features of the stream which are much less well understood, are probably of most value to the professional oceanographer. In particular, the discussion of meanders and fluctuations is suggestive rather than definitive. The reviewer feels that the usefulness of these sections is limited by the omission of theoretical calculations, the results of which are discussed at some length (e.g. the time-dependent problem of Veronis and Morgan and the stability analysis of Haurwitz and Panofsky). A full appreciation of Stommel's discussion can be achieved only by going to the original literature and then returning to the book.

Our knowledge of the thermohaline circulation of the oceans is advancing rapidly. The delay of four years between the writing and publication of the book has made this section already out-of-date, as may be inferred from the footnotes added in proof. The interested reader may wish to refer to two articles by Stommel and Arons, 'On the abyssal circulation of the world ocean' parts I and II, appearing this year in *Deep Sea Research*, which also contain references to relevant intervening work.

Throughout *The Gulf Stream* there persists a critical analysis of underlying assumptions and hypotheses which are made both in the interpretation of oceanographic observations and in the formulation of mathematical models. This constitutes a valuable contribution to oceanography. The book itself is attractively produced and the author's style is highly readable. The bibliography, data sources, diagrams and illustrations form a valuable collection. This short treatise is highly recommended to all scientists with a real curiosity concerning the Gulf Stream and the associated wealth of fascinating oceanographic phenomena.

ALLAN R. ROBINSON

Viscoelasticity: Phenomenological Aspects. Edited by J. T. Bergen. New York: Academic Press, 1960. 150 pp. $6.00.

This book consists of seven papers given at a symposium in Pennsylvania organized by the Armstrong Cork Company in April 1958. The subject is phenomenological or macroscopic viscoelasticity; that is, the viscoelastic material is treated as a continuum and its response to stress or strain is described by equations whose elements are derived from the theories of continuum mechanics. Each paper is an up-to-date treatment of a particular aspect of the subject. The book is not a comprehensive treatise but, in the reviewer's opinion, provides an excellent introduction to the concepts of the subject to anyone prepared to read it through at least twice.

The first three papers treat linear viscoelasticity: they are 'Stress analysis for viscoelastic bodies' by E. H. Lee, 'The linear viscoelastic behaviour of rubber-like polymers and its molecular interpretation' by Robert S. Marvin, and 'Comparisons of viscoelastic behaviour in seven typical polymer systems' by John D. Ferry and Kazuhiko Nimomiya. The last four papers are concerned with non-linear viscoelasticity: 'The behaviour of certain viscoelastic materials in laminar shearing motions' by J. L. Ericksen, 'Constitutive equations for classes of deformations' by R. S. Rivlin, 'Stress relaxation of polymeric material in combined torsion and tension' by J. T. Bergen, and 'The normal stress effect in polymer solutions' by Hershel Markovitz. (If any stress $\sigma_1(t)$ produces strain $\epsilon_1(t)$ and any stress $\sigma_2(t)$ produces strain $\epsilon_2(t)$, then the material is said to be linear if stress $\sigma_1(t)+\sigma_2(t)$ produces strain $\epsilon_1(t)+\epsilon_2(t)$, t being time.)

The analyses of stress and strain (or strain rate) are common to all branches of continuum mechanics. It is the set of relationships between stress and strain which characterizes the different branches of the subject. For an isotropic linear viscoelastic material there are two independent functional relationships, the one connecting each deviatoric component of stress to the corresponding deviatoric component of strain and the other connecting the first invariant of stress to the dilatation; both relationships can be expressed in various equivalent forms. It is these forms, the means of converting one into another and the molecular interpretation of the model form, which Marvin treats in his paper. Experimentally it is possible to determine the complex modulus, the ratio of stress to strain at angular frequency ω when both are expressed in complex form $A \exp(i\omega t)$, for frequencies in the range 10^{-2} to 10^{10} radians per second. The creep function, strain against time at unit constant stress, can be measured for times greater than 1 sec. There is an overlap of about two decades, and it is possible to use either set of measurements to complete the time spectrum of the other set in the range where actual experimental measurement is not possible. The long-time end of the scale can frequently be extended by using the experimental fact that for many materials an increase in temperature casues a bodily shift of the real part of the complex compliance-frequency curve. The reviewer is rather doubtful, however, whether Marvin's extrapolation of the relaxation function to 10^{10} years is valid. (Complex compliance is the reciprocal of complex modulus and the relaxation function is the stress response to unit constant strain.)

Ferry and Nimomiya give the basic deviatoric viscoelastic functions for seven typical polymer systems. Five of the systems are amorphous polymers (i) of low molecular weight, (ii) of high molecular weight, (iii) of high molecular weight with long side groups, (iv) lightly vulcanized, and (v) below the glass transition temperature; the remaining systems are (vi) dilute polymer gel, and (vii) highly crystalline polymer. This list provides examples of actual materials to which the theory of linear viscoelasticity can be applied. Little experimental work appears to exist on the determination of dilatational viscoelastic functions.

Lee's paper is concerned with the solution of boundary value problems for linear viscoelastic materials. The governing differential equations are the strain-displacement equations, the equations of motion and the stress-strain relationships. It is frequently convenient to subject the time variable to an integral transform. The Laplace transforms of the stress-strain relationships involve the Laplace transforms of the creep or of the relaxation functions, while the Fourier transforms involve the complex moduli or compliances. If the boundary conditions are such that their transforms exist, then the transform approach is generally the best for that particular problem. The inverse transformation in general has to be evaluated numerically, although Lee gives examples where the stresses can be evaluated as explicit functions of space and time for the relatively simple case of a Maxwell material. For other problems it may be necessary to approximate to the stress-strain relationship by an equation of the form $P\sigma = Q\epsilon$, where P and Q are polynomials with constant coefficients in the operator d/dt; the fewer terms in P and Q, the easier the problem is to solve but the less accurate in general is the approximation. It is essential that such an equation be fitted to the experimentally measured viscoelastic functions over the time range which is significant for the particular problem considered.

It can be seen from the foregoing paragraphs that the theory of linear viscoelasticity is fairly comprehensive. This is not true of the non-linear theory. There is considerable diversity of opinion as to the form of the stress-strain relationship in the general case; in fact it is incorrect to talk of strain in this context, since workers in this field formulate relationships between the stress and the time derivatives of the displacement gradients. At present progress is being made by finding the stress, not for general displacement of a limited number of materials, but for particular classes of displacement of a wide range of materials.

Erickson starts by considering the postulate that the stress in simple shearing motion ($v_1 = Kx_2$, $v_2 = v_3 = 0$) is a function of certain symmetric tensors which appear in formulae for successive time derivatives of the distance between any two neighbouring particles. The components of stress can then be expressed as functions, in general arbitrary, of K and the time. The relationship thus formulated can be extended to any flow, which has the property that, in the neighbourhood of each point at any time, the flow can be reduced to the form $v_1 = Kx_2$, $v_2 = v_3 = 0$ by a rotation of axes. The rotation matrix in general depends on space and time. Such flows are called laminar shear flows. An interesting application is to flow through cylindrical pipes; it is the form of the arbitrary functions which decides whether linear flow, without secondary flow, is possible for a particular liquid through a pipe of arbitrary cross-section.

Rivlin investigates the stress-displacement gradient relationship for fixed displacements and for times large compared to the time at which the displacement was completed. The relationship contains the time explicitly. The form of the relationship is limited by two restrictions: (i) isotropy, and (ii) if the deformed body is subject to a rigid rotation, then the stress field is subject to the same rotation. The relationship is further simplified if the gradients are small. Rivlin calculates the torque and tension on a thin tube held at constant torsion and extension; the results contain one arbitrary function, a function of time and two invariants of the displacement gradient tensor.

Anyone who feels that Rivlin's paper is no more than an agreeable exercise in pure mathematics will be disillusioned by the paper that follows. Bergen carries out the experiments suggested by Rivlin on thin tubes of vulcanized rubber and of a type of polyvinyl chloride composition used for floor coverings. He finds that the arbitrary function mentioned above is of the form $\alpha(t)\beta(J)$, where J is the determinant of the classical strain matrix; he determines the functions α and β for these materials. In addition, by varying the rate at which the extension (without torsion) was initially applied, Bergen verifies Rivlin's initial hypothesis for these materials: if the time taken to apply the constant extension is T, then Bergen finds that the tension is independent of the rate of application at times later than $7T$ after the extension has become constant.

In the simple shearing motion considered by Ericksen, the normal deviatoric stress σ'_{22} is non-zero for most polymer solutions. Markovitz measures this stress experimentally with parallel plates, with cone and plate and with coaxial cylinders for various polymer solutions. He compares the measured stresses with those predicted by various rheological theories; the results are inconclusive.

D. R. Bland

Hypersonic Flow. Edited by A. R. Collar and J. Tinkler. Butterworth Scientific Publications, 1960. 70*s*.

This volume records the Proceedings of the Eleventh Annual Symposium of the Colston Research Society held in the University of Bristol in April 1959. The interests of the Colston Society are wide and varied and in past years subjects for symposia have been chosen in science and the arts, in medicine and administration; in 1959 Hypersonic Flow was chosen as an appropriate theme as this field of fluid dynamics is at present undergoing an interesting and active period of development.

Hypersonic conditions of flow correspond roughly to flows in excess of Mach number 5 and usually also imply high stagnation temperatures. In recent years the study of such flows has become important because of the rapid developments in rocket propulsion; such conditions obtain with long-range missiles, with artificial satellites and with space probes, particularly during the later stages of their launching period or during their re-entry into the Earth's atmosphere.

This meeting was attended by about one hundred members. Fifteen papers were presented and discussed at six half-day sessions. Five of the papers were of a theoretical nature, and seven papers described a range of experimental facilities and recent results obtained in them; the remaining three papers dealt with design and operational aspects of hypersonic vehicles and propulsive units.

In the theoretical field, Guiraud of ONERA presents an extended examination of the Newtonian approximation when applied to the study of three-dimensional hypersonic flows. He derives formulae for the axial and normal forces and for the moment acting on a general body, and then more particularly for a cone and for a body of revolution at an angle of attack. The existence of a Ferri vortical layer on the surface of such a body is touched upon. Van Dyke of NASA describes some numerical methods of calculating the flow near the nose of a blunt body and in the neighbourhood of non-circular cones; marching techniques are used in both examples, starting from an assumed shock wave surface and terminating on the solid surface producing the disturbance. His results are generally satisfactory and compare well with similar results obtained by other investigators using slightly different methods. Mangler of the Royal Aircraft Establishment also describes a method of investigating the flow around a blunt body by a marching technique using specially chosen independent variables and describes some results obtained on an electronic computer. Miles of the University of California, Los Angeles, surveys methods of dealing with problems of unsteady flow at hypersonic speeds and examines various systems of approximation; he concludes that an adequate approximation to the unsteady aerodynamic pressure p' acting on bodies executing small motions in a hypersonic stream may be calculated from the plane wave relation $p' = \rho a v'$, where it is assumed that changes in entropy associated with the unsteady motion may be neglected. v' is the perturbation velocity normal to the perturbation wave front. In the fifth theoretical paper, Spence and Woods of the Royal Aircraft Establishment examine boundary layer and combustion effects that arise in a combustion-driven shock tube.

Of the seven papers dealing with experimental techniques and results, three describe recent work at the National Physical Laboratory. Henshall presents an account of measurements of shock speed and running time of their main shock tunnel; he also records measurements of the rate of heat transfer near a stagnation point and touches upon an investigation into the starting processes of the tunnel. In another paper Schultz describes investigations of gas ionization by an application of microwave techniques. Lapworth summarizes similarly recent results on the spectrograph measurement of gas temperatures in shock-tube flow for ranges of temperature below 10,000° K. Cox of the Armament Research and Development Establishment describes the development of light gas-gun tunnels at Fort Halstead; one of the main features of these tunnels is their longer running time when compared with that of the normal shock tube. He also touches upon the techniques used in these tunnels for measurements of pressure distributions, forces, heat transfer, etc., and describes a number of investigations designed to provide a comparison with other facilities. Bray of the University of Southampton reports a series of investigations designed to study the performance of their light gas-gun tunnel in considerable detail. This work includes the recording of the motion of the piston, an analysis of the shock reflexion process and of the expansion conditions in the nozzle. Lobb of the U.S. Naval Ordnance Laboratory describes the expansion of the hypersonic test facilities at White Oak during recent years. This equipment includes wind tunnels, shock-tube tunnels and ballistic ranges. He also surveys the measuring techniques currently in use and presents a variety of results on real gas effects, on boundary-layer flow and on aerodynamic forces. Current hypersonic research at the Franco-German Institute (ISL) at Saint Louis is described by Oertel. This work includes the application of ballistic range techniques when carbon dioxide or carbontetrachloride is the working medium; use is also made of a shock expansion tube to obtain hypersonic flow of the order $M = 8$.

In the design field Eggers of NASA examines how future aircraft configurations, with adequate pay-load, can be made to attain intercontinental and global ranges and thus exploit the advantages of hypersonic speeds. He bases his investigations on the boost-glide form of flight path. His conclusions are that with increasing speed and range the trend will be first to more slender configurations with higher lift/drag ratios but that, for semi-global ranges and for speeds approaching that of satellites, the configurations should be more blunt and have higher lift and drag. Metcalfe of Bristol Aircraft also describes vehicle configurations for a number of flight conditions and for a range of operational tasks. He touches upon several problems of aerodynamic control. In the third paper of this group Jamison of Bristol Siddeley Engines surveys the problems of air-breathing propulsion devices for hypersonic flight and concludes that the maximum overall efficiency of a hydro-carbon fuelled ram-jet should be realized at around Mach 7. He outlines a project study of an aircraft designed for this speed and having a great circle range of 5000 miles.

Research on hypersonic flow has been built up steadily in the U.S.A. over the last 10 or 15 years, and in the United Kingdom mainly over the last 6 years. Although it does not aspire to give a comprehensive account of all the fields of activity in the U.S.A., the present volume does give a very adequate survey of most of the current lines of research in Western Europe. The present group of papers indicates a marked increase in research activity in Europe when compared with the work described at a similar symposium organized by AGARD at Scheveningen in July 1957. The proceedings of these two meetings, when studied alongside a systematic volume, such as that of Hayes and Probstein (*Hypersonic Flow Theory*, Academic Press, 1959) furnish a very good and broad introduction to this relatively new subject.

J. W. MACCOLL